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A method of determining the shear modulus and Poisson's ratio of polymer materials

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An experimental method is described which allows the shear modulus and Poisson's ratio of a polymer material to be determined as functions of temperature. Using commercial thermal mechanical analysis equipment, measurements are required of the thermal expansion coefficients of both unconstrained and constrained specimens, and of the Young's modulus. The constraint on a small disc-shaped polymer specimen is applied by a copper annulus. The basis of the method is that the apparent thermal expansion of the constrained specimen depends on the Poisson effect, and an analysis is given which enbles the Poisson's ratio to be determined. Some illustrative results are presented for a silicon rubber.

1. Introduction

The thermal expansion and Young's modulus of a polymer material can conveniently be measured over a range of temperatures using commercial thermal mechanical analysis (TMA) equipment. However, it is far more difficult to determine with much accuracy the variation of the shear modulus with temperature, and a simple method is described here for doing so which utilizes the results of conventional TMA equipment. The dependence of the Poisson's ratio of the material on temperature is obtained in this method.

The origins of the work reported here lay in a need to measure the thermal expansion coefficient and Young's modulus of elastomer materials to be used as tools for the fabrication of carbon fibre reinforced plastic components, either in the form of pressure bags [1] or as thermal expansion moulds [2]. In the latter application, the expansion of the mould is constrained by an external metal box, and it was therefore of interest to determine the effective thermal expansion coefficient of the material when it was constrained in two dimensions. These measurements were made with a Mettler TMA unit and, as shown in detail below, a knowledge of the linear thermal expansion coefficient of the constrained material allows the Poisson's ratio of the material to be determined. A further measurement of the Young's modulus allows the shear modulus to be obtained using a standard relationship between the elastic constants of linearly elastic materials.

2. Thermal expansion under constraint

Commercial thermal mechanical analysis equipment typically uses specimens of polymers which are disc-shaped, with a diameter of several millimetres and with a thickness of a few millimetres. Measurement of the changes in thickness of the specimen with increasing temperature enables the linear coefficient of thermal expansion, α, to be determined. A convenient method of constraining the material in the plane of the disc is to surround it by a ring of a material with much lower thermal expansion, for example a section cut from a metal tube of suitable diameter, as illustrated in Fig. 1. Since reaction stresses are exerted on the circumference of the specimen as the temperature is raised, an extra elongation of the specimen perpendicular to the disc will be produced due to the Poisson effect. If it is assumed that the annular thickness of the constraining ring is small, and that the polymer is isotropic and behaves elastically, the effects of the constraint can readily be analysed.

It is convenient to use cylindrical co-ordinates, as illustrated in Fig. 1. In the unconstrained case the thermal strain, ε_z, in a direction perpendicular to the disc, resulting from a temperature rise, ΔT, is

$$\varepsilon_z = \alpha\Delta T \tag{1}$$

This is the quantity measured.

In the constrained case, the radial expansion of the polymer is restricted by the surrounding sleeve. The strains produced are partly due to thermal expansion and partly due to the reaction stress, components σ_r, σ_θ and σ_z. Denoting Poisson's ratio by ν and Young's modulus by E, and assuming that Hooke's law applies, then [3]

$$\begin{aligned}\varepsilon_r - \alpha\Delta T &= [\sigma_r - \nu(\sigma_\theta + \sigma_z)]/E \\ \varepsilon_\theta - \alpha\Delta T &= [\sigma_\theta - \nu(\sigma_z + \sigma_r)]/E \\ \varepsilon_z - \alpha\Delta T &= [\sigma_z - \nu(\sigma_r + \sigma_\theta)]/E\end{aligned} \tag{2}$$

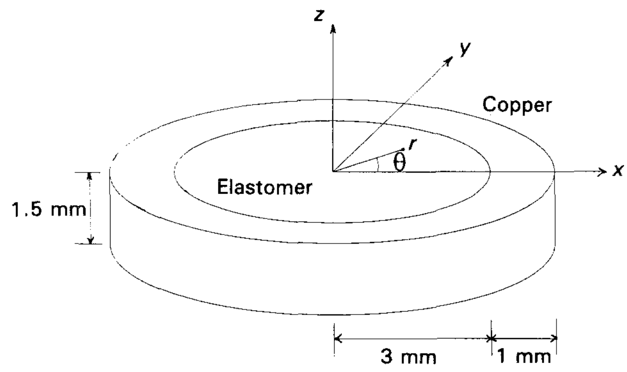

Figure 1 Elastomer disc specimen constrained by a copper ring.

Because the specimen is unconstrained in the z-direction, the situation is one of plane stress, such that

$$\sigma_z = 0 \quad (3)$$

It will be assumed σ_r and σ_θ are independent of z, and it can then be shown [3] that for a uniform temperature distribution

$$\sigma_r = \sigma_\theta \quad (4)$$

The radial strain of the specimen on its circumference is determined by the radial expansion of the restraining ring,

$$\varepsilon_r = \alpha_0 \Delta T \quad (5)$$

where α_0 is the linear thermal expansion coefficient of the ring. From the first of the Equations 2, noting Equations 3 and 4, it follows that the stress exerted on the disc has components

$$\sigma_r = \sigma_\theta = \frac{(\alpha_0 - \alpha)\Delta T E}{(1 - \nu)} \quad (6)$$

Substitution into the third of the Equations 2, again noting Equation 3, yields for the strain in the z-direction

$$\varepsilon_z = \left[\frac{1 + \nu}{1 - \nu}\right]\alpha \Delta T - \frac{2\nu}{(1 - \nu)}\alpha_0 \Delta T \quad (7)$$

Experimental measurements of the strain are conveniently expressed in terms of an effective constrained linear thermal expansion coefficient, α_c say, defined by

$$\varepsilon_z = \alpha_c \Delta T \quad (8)$$

Comparison with Equation 7 indicates that

$$\alpha_c = \left[\frac{1 + \nu}{1 - \nu}\right]\alpha - \frac{2\nu}{(1 - \nu)}\alpha_0 \quad (9)$$

Measurements of $\alpha(T)$ and $\alpha_c(T)$ as functions of temperature enable Poisson's ratio $\nu(T)$ to be determined as a function of temperature, provided that the expansion property of the ring material, i.e. $\alpha_0(T)$, is known. Rearranging Equation 9, Poisson's ratio is given by

$$\nu = \frac{(\alpha_c - \alpha)}{(\alpha_c + \alpha - 2\alpha_0)} \quad (10)$$

The determination of the shear modulus, G, as a function of temperature requires a knowledge of the Young's modulus $E(T)$. The latter may be obtained from separate measurements on an unconstrained specimen using commercial TMA equipment. Assuming that the polymer has linearly elastic properties, both the shear modulus and the bulk modulus, K, can be evaluated using standard relationships [4] between the elastic constants

$$G = \tfrac{1}{2}E/(1 + \nu) \quad (11)$$

$$K = \tfrac{1}{3}E/(1 - 2\nu) \quad (12)$$

3. Results for a silicon elastomer

As an illustrative example of the use of the above methods, some experiments were made on specimens of a silicon RTV elastomer. Measurements of the linear expansion coefficients under unconstrained and constrained conditions, and of the Young's modulus, were made using a Mettler TA3000 thermomechanical analyser controlled by a TM4000 microprocessor. Disc-shaped specimens of 6 mm diameter were cut from sheets of the polymer approximately 1.5 mm thick. The temperature range used was 25–175 °C, and the heating rate was 2 °C min^{-1}. For the determination of the thermal expansion coefficients, a static load of 0.1 N was applied to the specimen, and for measurements of the Young's modulus an oscillatory force of amplitude 0.05 N was superimposed on the static force. The constrained condition was achieved by enclosing a polymer specimen with a copper ring of depth 1.5 mm and wall thickness approximately 1 mm. The linear thermal expansion coefficient of copper was measured using a disc-shaped specimen. The results obtained for copper indicated that the coefficient is almost constant over the temperature range of interest and has a value of $17 \times 10^{-6}\,°\text{C}^{-1}$, in good agreement with the accepted value.

The results obtained for the linear coefficients of expansion of the elastomer are displayed in Fig. 2. As would be expected, the effect of constraint is to increase the effective expansion. Fig. 3 shows that the Young's modulus is almost independent of temperature. The Poisson's ratio was determined using Equation 10 and the results are presented in Fig. 4. The ratio increases markedly with temperature between 25

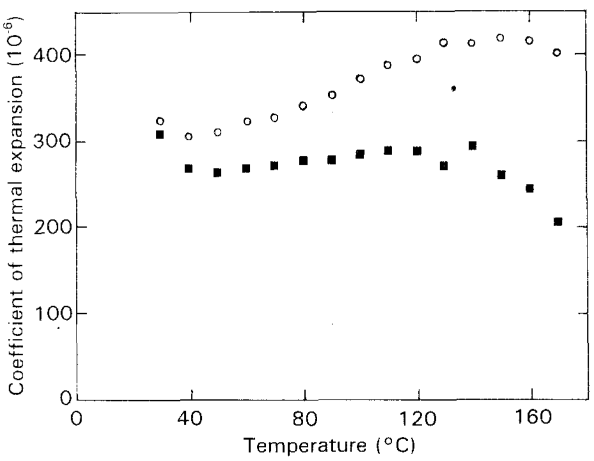

Figure 2 Linear coefficient of thermal expansion for a silicone elastomer: (■) unconstrained specimen, (○) constrained specimen.

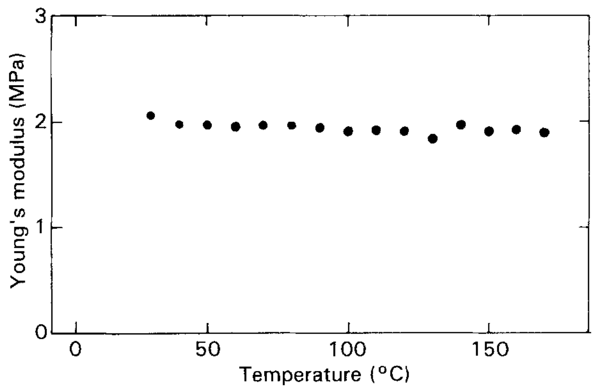

Figure 3 Young's modulus for the silicone elastomer.

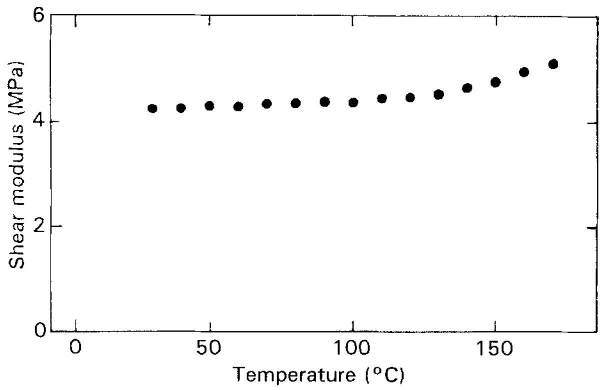

Figure 5 Shear modulus for the silicone elastomer.

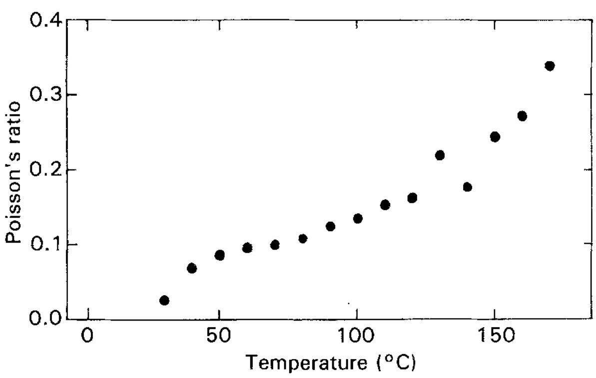

Figure 4 Poisson's ratio for the silicone elastomer.

and 175 °C. Finally, the shear modulus was obtained using Equation 11, and the values obtained are displayed in Fig. 5. For this particular polymer, over the temperature range studied, the shear modulus varies very little with temperature.

4. Discussion

The result presented above illustrate how well the method works for determining the shear modulus of a polymer. It has been assumed that the elastic properties of the material are linear, and this is normally valid provided that the strains are small. Although three separate measurements are required, these are usually routine and straightforward.

The method is being applied by the authors to a range of materials, some of whose elastic properties vary with their thermal history. The results of these further experiments will be applied to calculate the compression stresses of expansion moulding tools, and this work will be reported in the future.

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